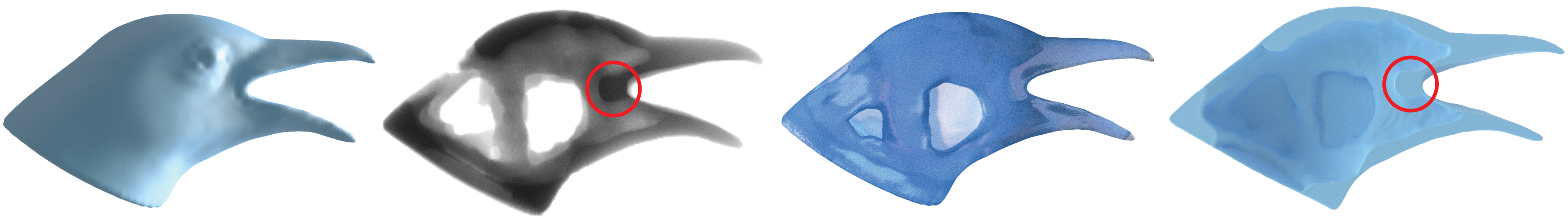
Fast and Robust Stochastic Structural Optimization



Qiaodong Cui[1] Timothy Langlois[2] Pradeep Sen[1] Theodore Kim[3]

University of California, Santa Barbara[1] Adobe Research[2] Yale University[3]

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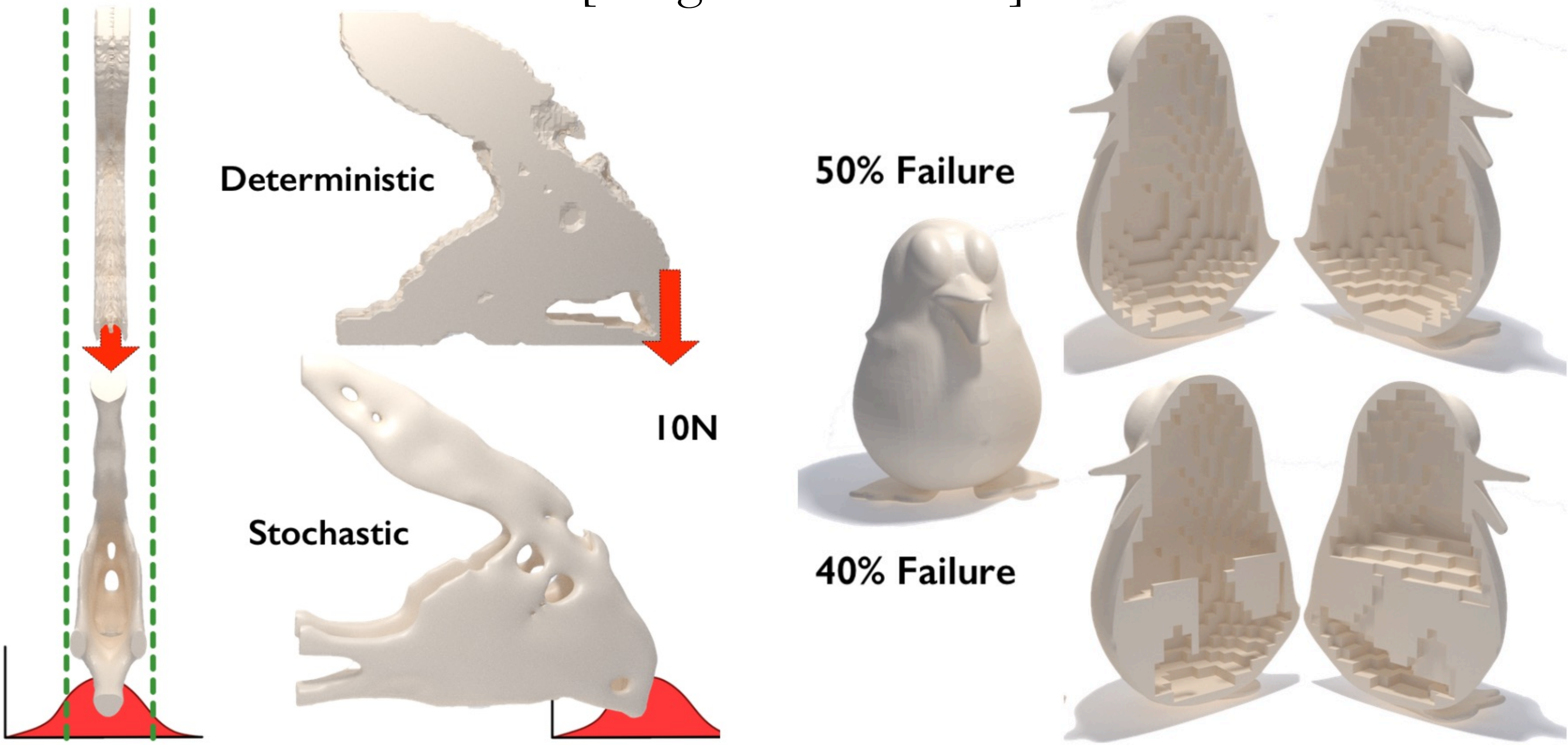
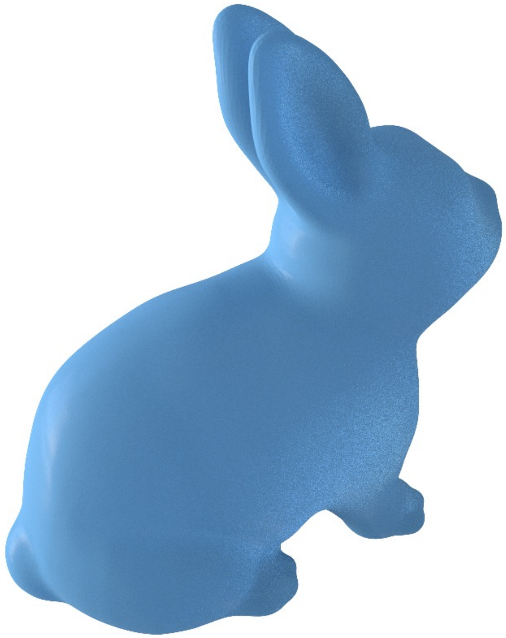
Stochastic Structural Optimization

[Langlois et al. 2016]

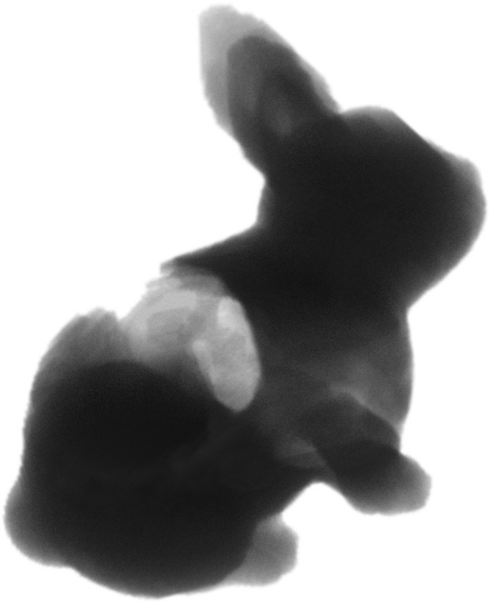


$26 \times 34 \times 28$ ~ 1 hrs per iteration

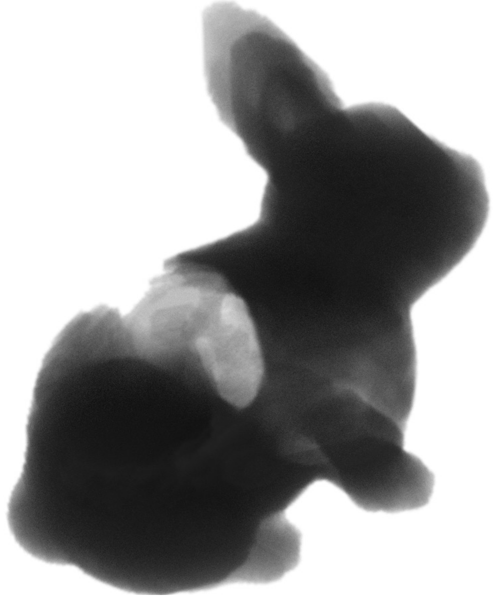
Initial shapes:



[Langlois et al.]

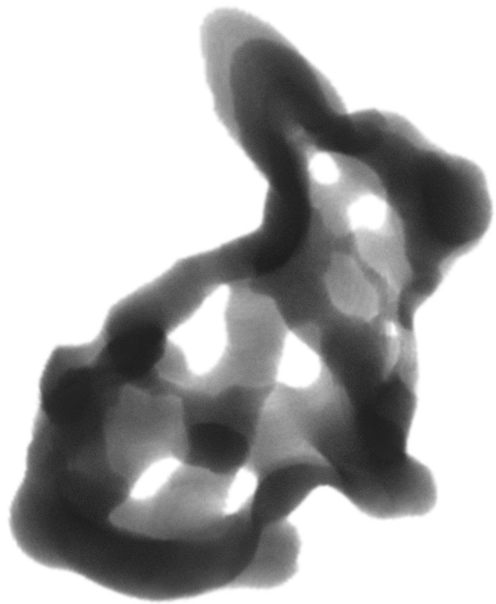


[Langlois et al.]



~ 6 hrs per iteration
97.3 g

Ours



~ 2.4 minutes per iteration
36.7 g

$28 \times 44 \times 28$

Outline

- Previous work
- Stochastic Structural Optimization
- Our methods
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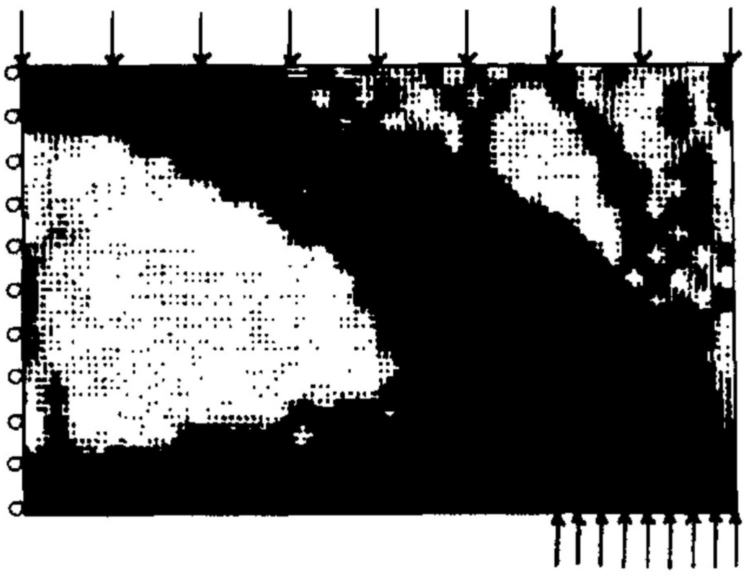
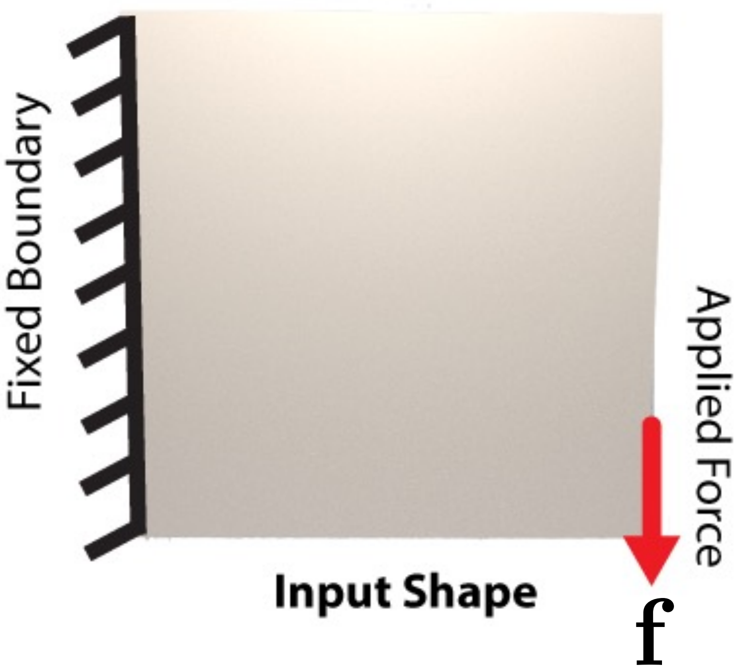
Structure Optimization

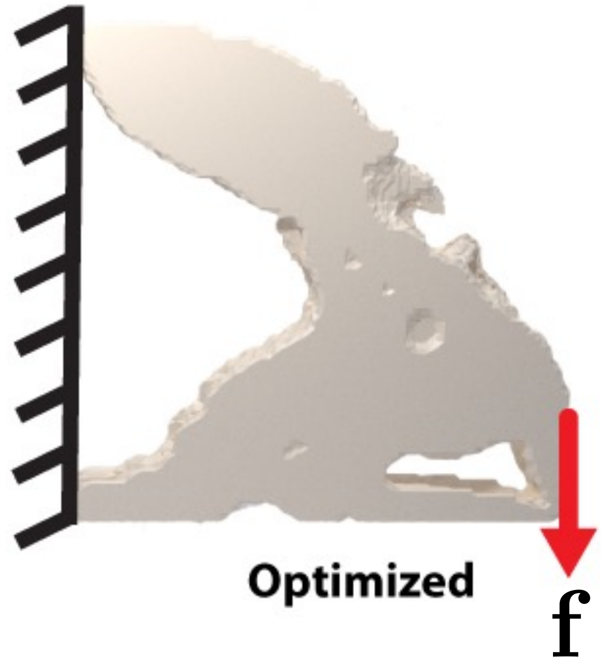


[Bendsoe et al. 1989]

[Wang et al. 2013]

Structure Optimization





$$\mathbf{u} = \mathbf{K}^{-1}\mathbf{f}$$

$\mathbf{f}$ is fixed

Compliance: $\mathbf{u}^T\mathbf{K}\mathbf{u}$

Minimization: $\min_{\boldsymbol{\omega}} \mathbf{u}^T\mathbf{K}\mathbf{u}$

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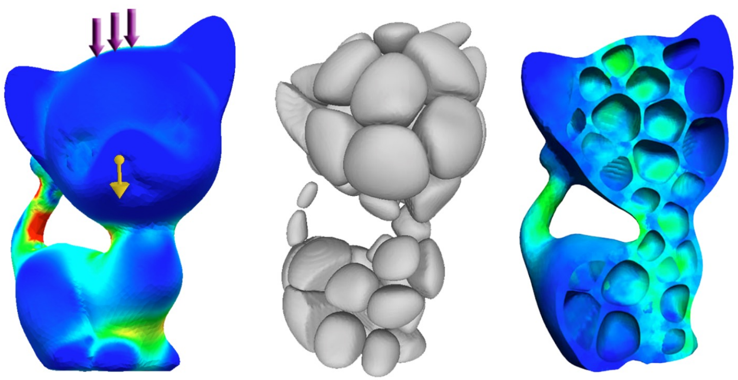
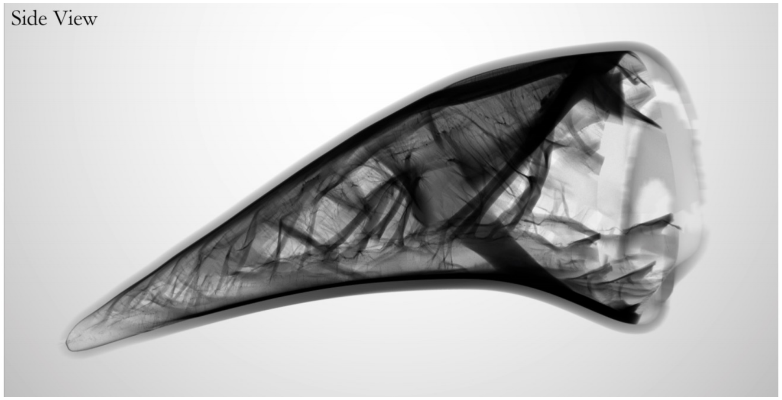
Structure Optimization

Compliance Minimization

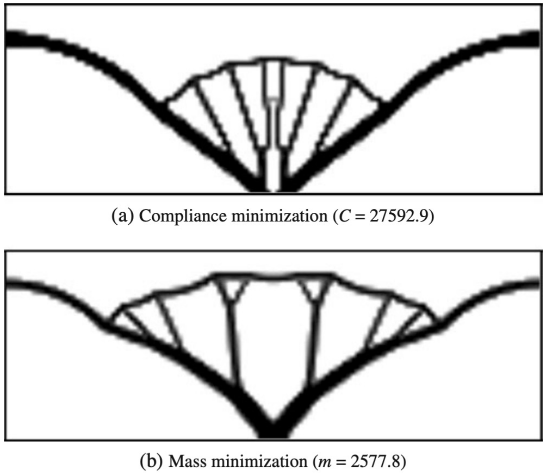


[Lu et al. 2014]

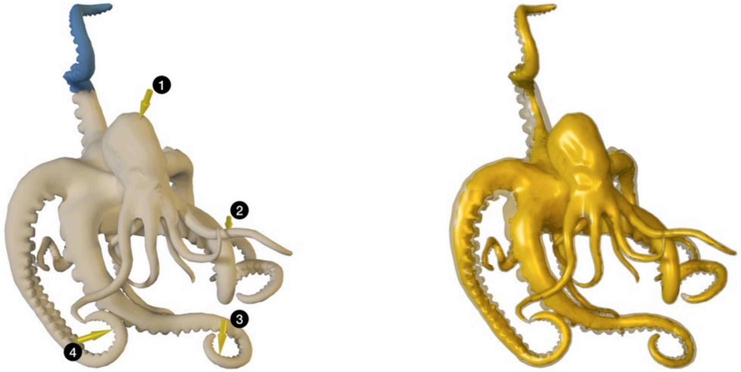


[Liu et al. 2018]

Weight Minimization



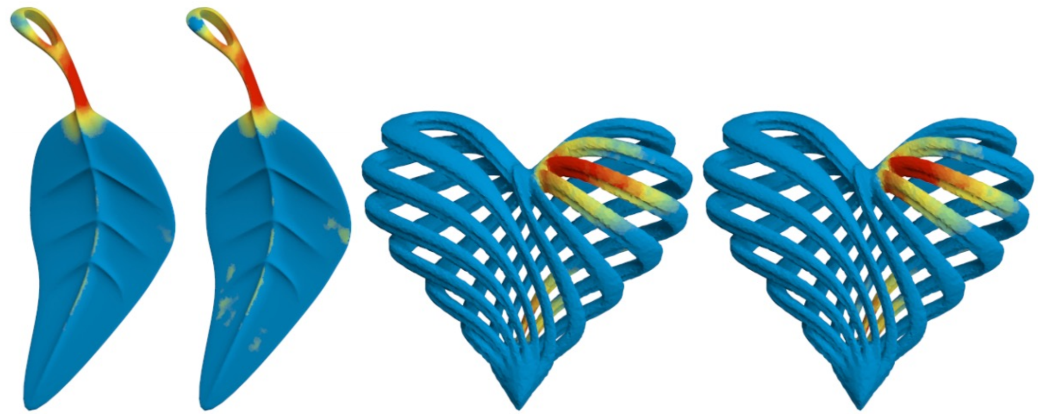
[Lee et al. 2012]



[Ulu et al. 2018]

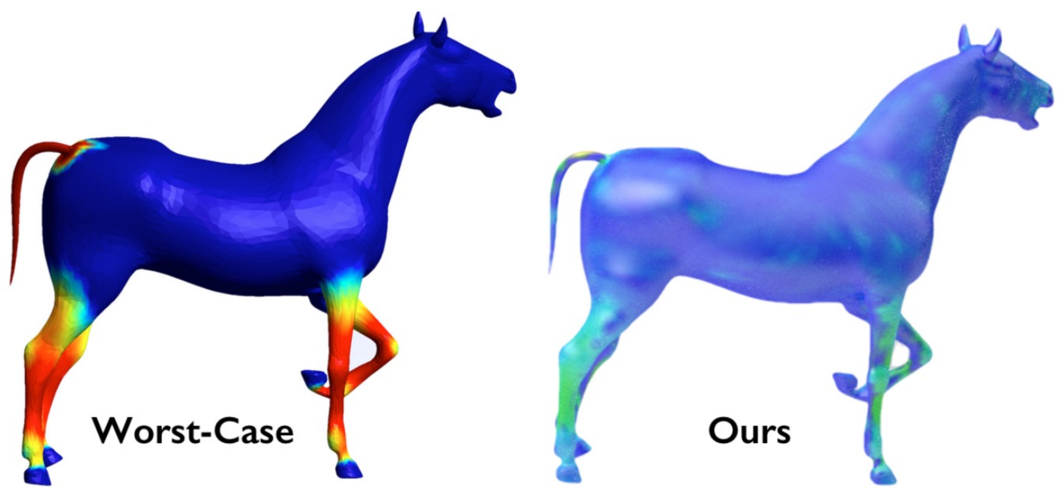
Failure Analysis

Worse Case
Structure Analysis



[Zhou et al. 2013]

Stochastic Case
Structure Analysis

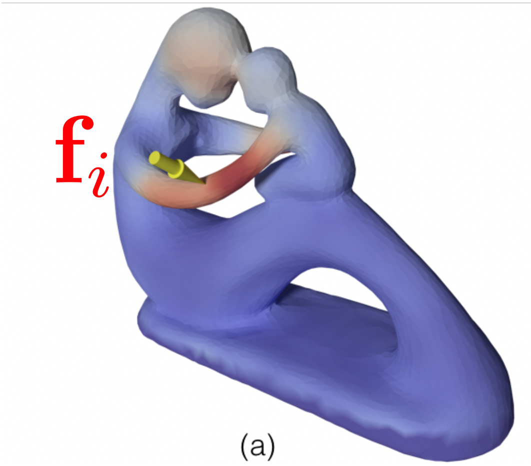


[Langlois et al. 2016]

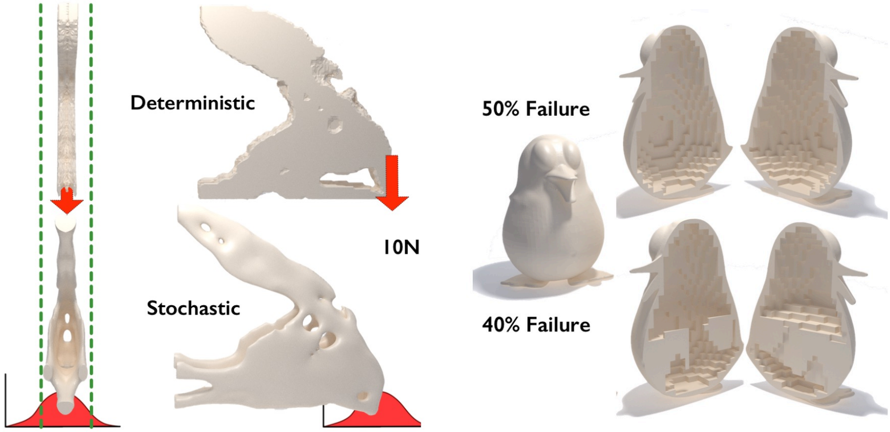
Structure Optimization

Worse Case Structure Optimization

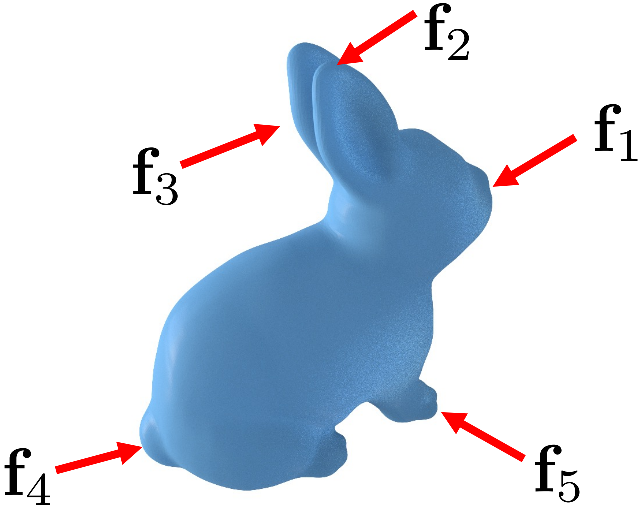
[Ulu et al. 2017]



Stochastic Structure Optimization



[Langlois et al. 2016]



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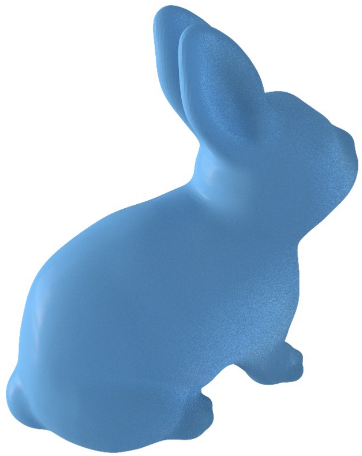
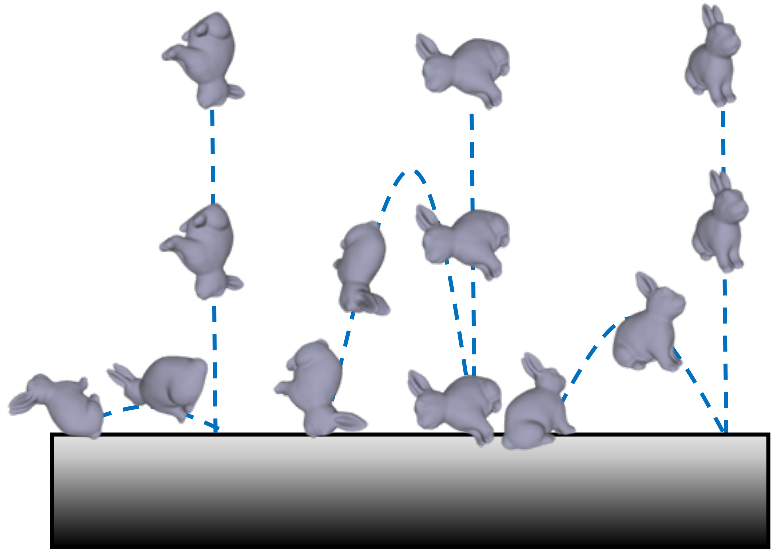
Stochastic Structural Optimization

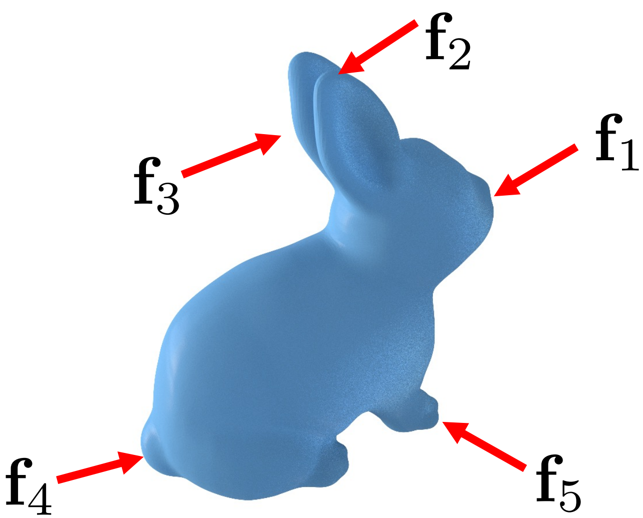


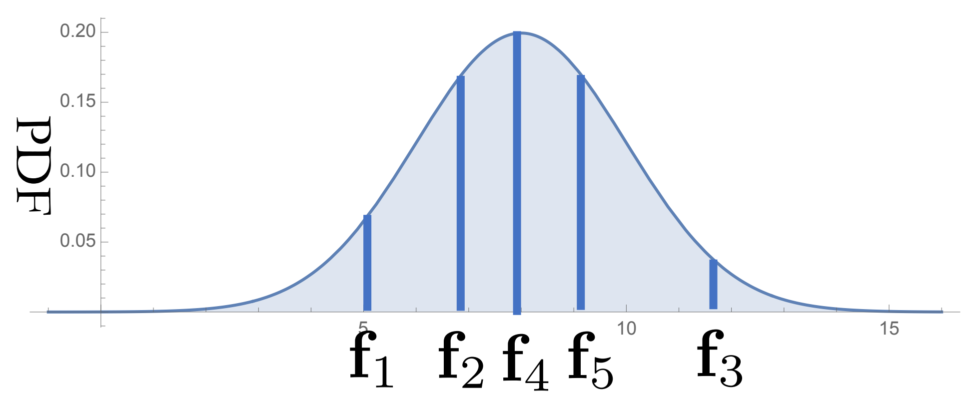
Initial Design



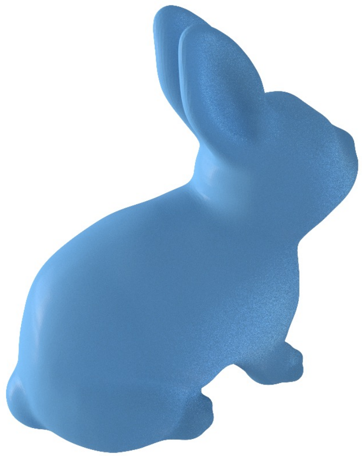
Rigid body simulation

Force sample: $\mathbf{f}_i$

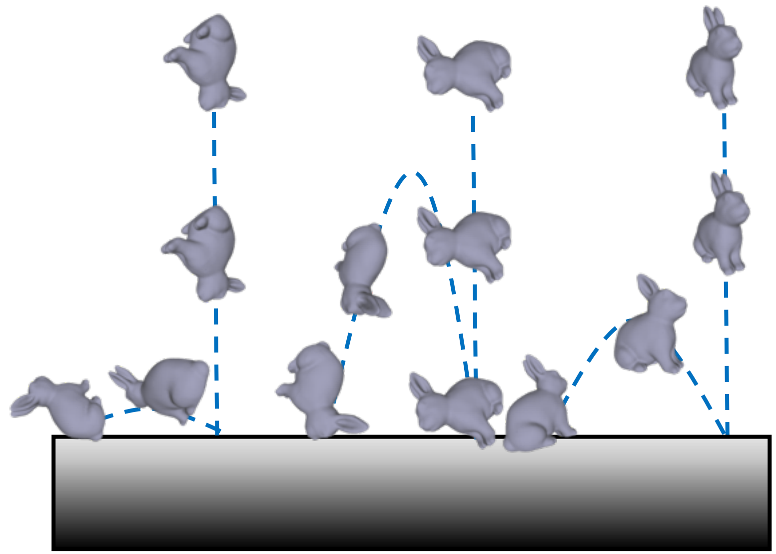




Stochastic Structural Optimization



Initial Design

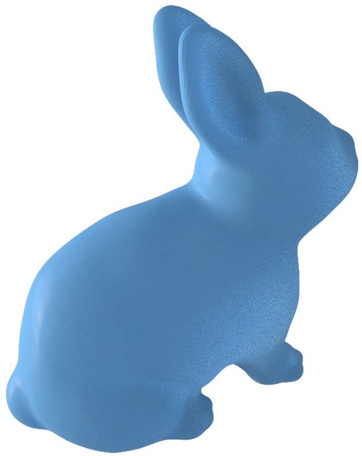


Rigid body simulation

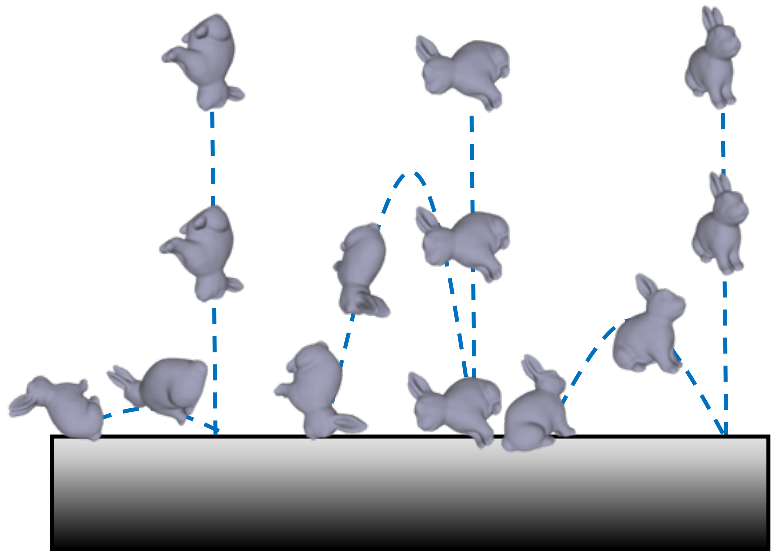
Force sample: $\mathbf{f}_i$

Force matrix: $\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 \ldots & \mathbf{f}_{n_s} \end{bmatrix}$

Stochastic Structural Optimization

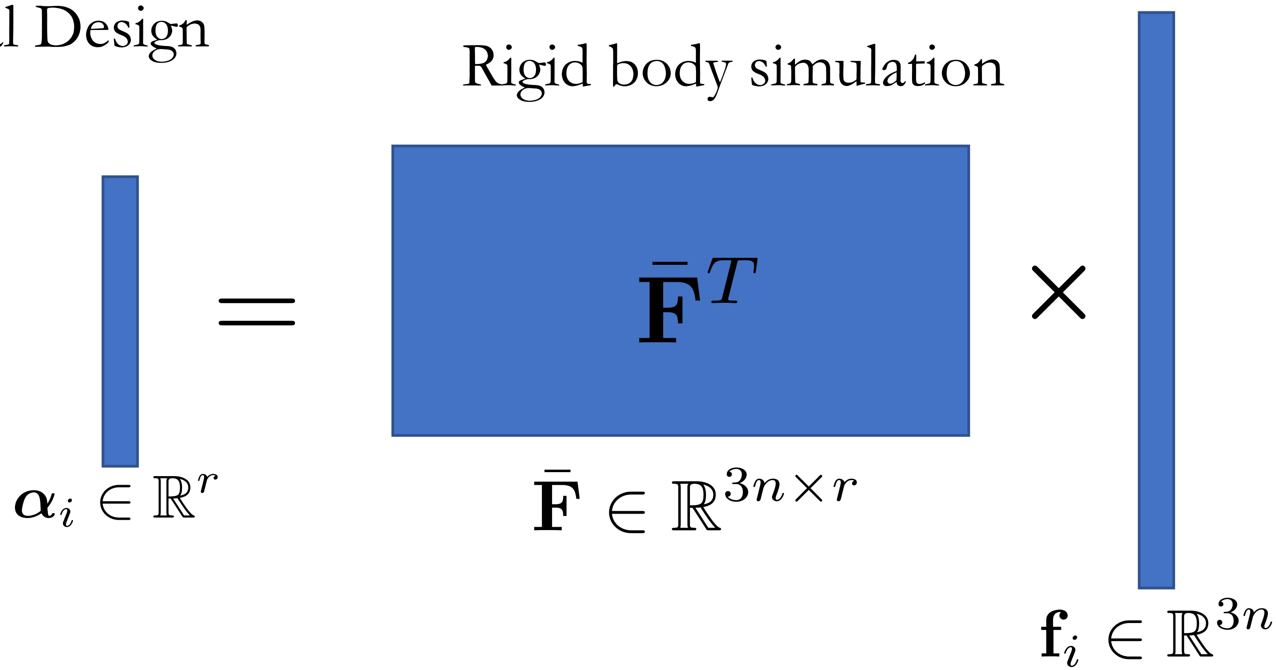


Initial Design

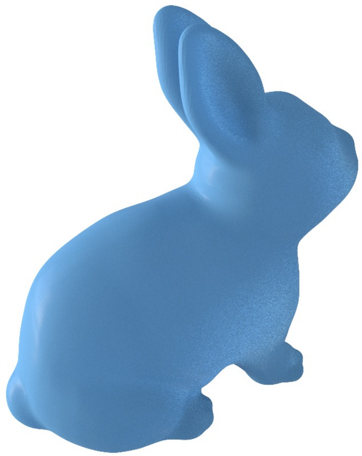


Rigid body simulation

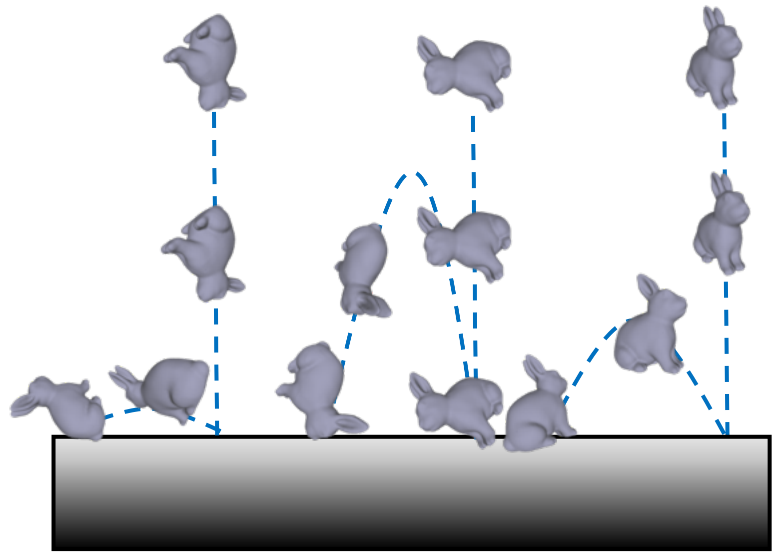
Force sample: $\mathbf{f}_i$



Stochastic Structural Optimization



Initial Design



Rigid body simulation

Force sample: $\mathbf{f}_i$

Force matrix: $\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 \dots & \mathbf{f}_{n_s} \end{bmatrix}$ $\quad \mathbf{K} \in \mathbb{R}^{3n \times 3n}$

PCA: $\mathbf{f}_i \approx \bar{\mathbf{F}} \boldsymbol{\alpha}_i$ $\quad \bar{\mathbf{F}} \in \mathbb{R}^{3n \times r}$

FEM solver: $\boldsymbol{\sigma}^i = \mathbf{CBK}^{-1} \bar{\mathbf{F}} \boldsymbol{\alpha}^i$ $\quad \boldsymbol{\alpha}^i \in \mathbb{R}^r$

Stochastic Structural Optimization

Maximum Von Mises Stress:

$$s^i = \frac{1}{\hat{\sigma}} \max_e(S(\boldsymbol{\sigma}_e^i)) \begin{cases} e = 1 \ldots m \\ i = 1 \ldots n_s \end{cases}$$

Probability of survival:

$$P(s < 1) = \int_0^1 p(s)ds.$$

Optimization Criteria:

$$\min \sum_{e=1}^{m} \omega_e$$

$$\text{s.t. } P(s < 1) > \Theta.$$

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  - Faster gradients computations
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Previous Method

- Gradient based optimization: Method of Moving Asymptotes (MMA) is used.

$$\frac{\partial P(s<1)}{\partial \boldsymbol{\omega}} = (\mathbf{K}^{-1}\mathbf{Y}\bar{\mathbf{U}}^{T}) : \frac{\partial \mathbf{K}}{\partial \boldsymbol{\omega}} + (\mathbf{K}^{-1}\mathbf{Y}) : \frac{\partial \bar{\mathbf{F}}}{\partial \boldsymbol{\omega}} + \mathbf{x} + \mathbf{t}$$

Force Basis Derivative:

$$\frac{\partial \bar{\mathbf{F}}}{\partial \boldsymbol{\omega}_e} = \bar{\mathbf{F}}\mathbf{W}_e$$

$$\mathbf{W}_e \in \mathbb{R}^{r\times r}$$

$$\bar{\mathbf{F}} \in \mathbb{R}^{3n\times r}$$

$\mathbf{W}_1$	$\mathbf{W}_2$	$\mathbf{W}_3$	$\cdots$	$\mathbf{W}_e$	$\cdots$

Previous Method

- Naïve evaluation is quadratic

Matrix production: $\bar{\mathbf{F}}\mathbf{W}_e \rightarrow O(3n^2r^2) \approx O(n^2r^2)$

$$\mathbf{Z} = \mathbf{K}^{-1}\mathbf{Y} \in \mathbb{R}^{3n\times r}$$

Matrix contraction: $\mathbf{Z} : \bar{\mathbf{F}}\mathbf{W}_e \rightarrow O(3n^2r) \approx O(n^2r)$

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Our Linear Method

Static per element: $\mathbf{Z}, \ \bar{\mathbf{F}}$

Varying per element: $\mathbf{W}_e$

$$\mathbf{Z} : \bar{\mathbf{F}}\mathbf{W}_e = (\bar{\mathbf{F}}^T\mathbf{Z}) : \mathbf{W}_e$$

Precompute: $(\bar{\mathbf{F}}^T\mathbf{Z}) \in \mathbb{R}^{r \times r} \longrightarrow O(3nr^2) \approx O(nr^2)$

Contraction : $(\bar{\mathbf{F}}^T\mathbf{Z}) : \mathbf{W}_e \longrightarrow O(nr)$

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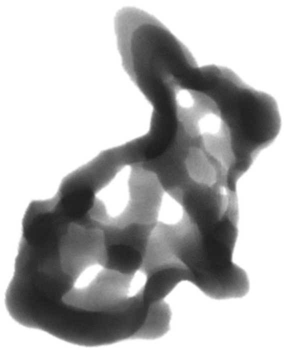
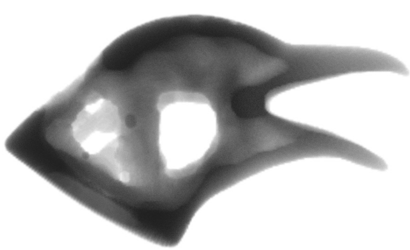
Our Linear Method



$28 \times 44 \times 28$



$32 \times 64 \times 40$

	[Langlois et al. 2016]	Ours	
$28 \times 44 \times 28$	~ 6.0 hrs per iteration	~ 2.4 minutes per iteration	150×
$32 \times 64 \times 40$	~ 11.8 hrs per iteration	~ 5.4 minutes per iteration	131×

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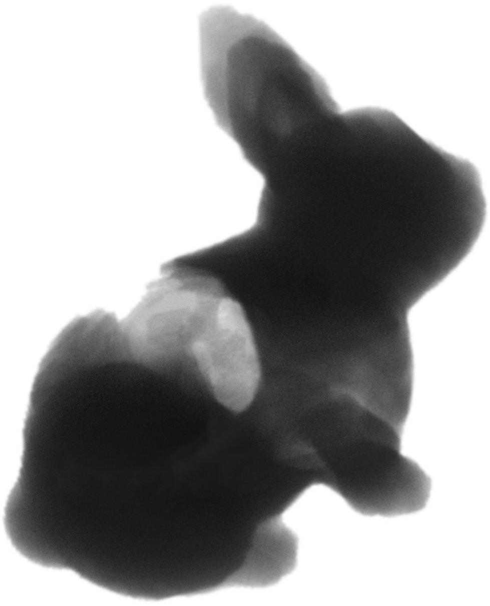
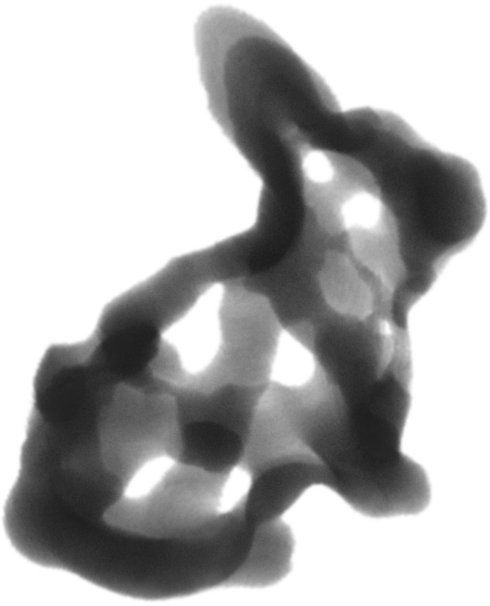
Instabilities

[Langlois et al. 2016]



Ours



Inertia Gradients

$$\frac{\partial P(s<1)}{\partial \boldsymbol{\omega}} = (\mathbf{K}^{-1}\mathbf{Y}\bar{\mathbf{U}}^T) : \frac{\partial \mathbf{K}}{\partial \boldsymbol{\omega}} + (\mathbf{K}^{-1}\mathbf{Y}) : \frac{\partial \bar{\mathbf{F}}}{\partial \boldsymbol{\omega}} + \boxed{\mathbf{x}} + \mathbf{t}$$

$$\mathbf{x} = \sum_{i=1}^{n_s} \boxed{\frac{\partial \boldsymbol{\alpha}^i}{\partial \boldsymbol{\omega}}} \bar{\mathbf{U}}^T \mathbf{B}^T \mathbf{C}^T \mathbf{c}^i \qquad \boldsymbol{\alpha}^i \in \mathbb{R}^r$$

$\frac{\partial \boldsymbol{\alpha}^i}{\partial \boldsymbol{\omega}}$ are evaluated with finite difference by computing a probability distribution function of $\boldsymbol{\alpha}^i$

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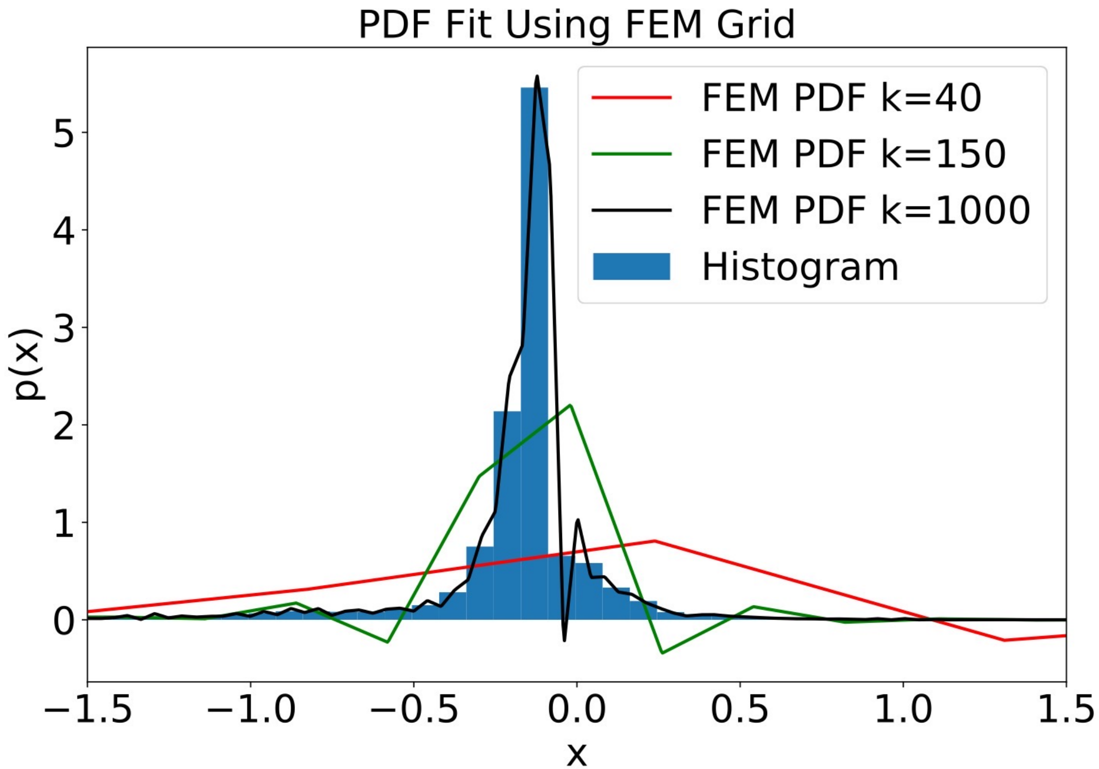
Unstable Gradients

A 1D probability distribution function $c_j(\alpha)$ is computed for each entry j, $1 \leq j \leq r$, using n_s samples.



[Langlois et al. 2016]

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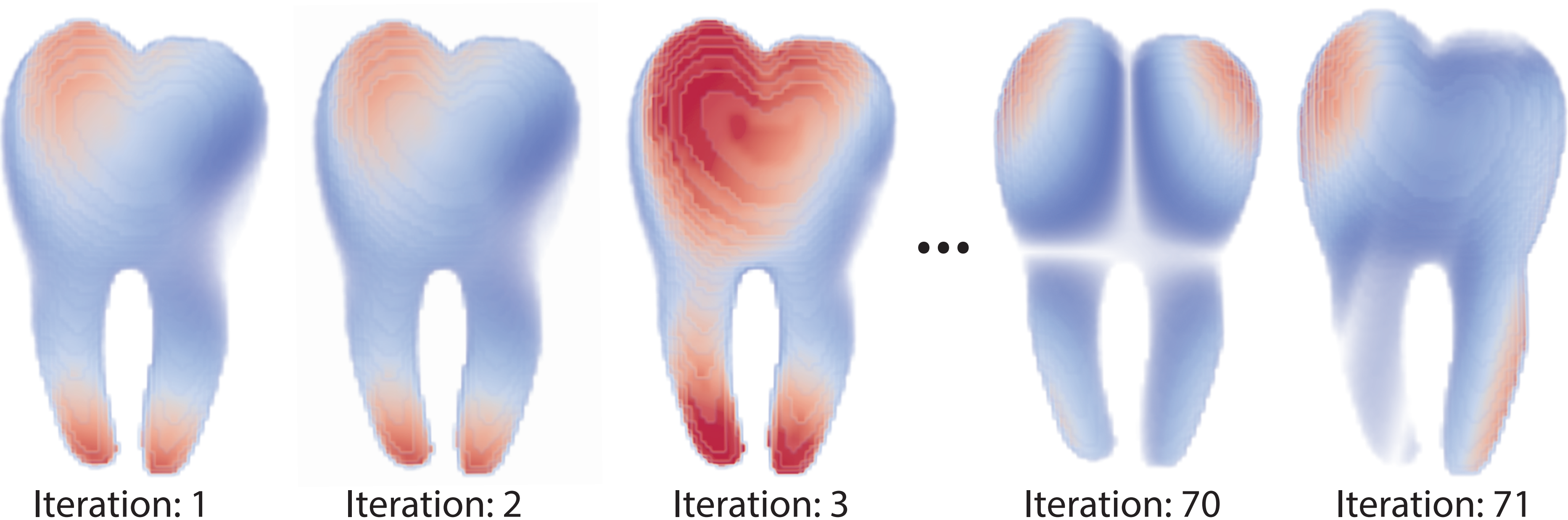
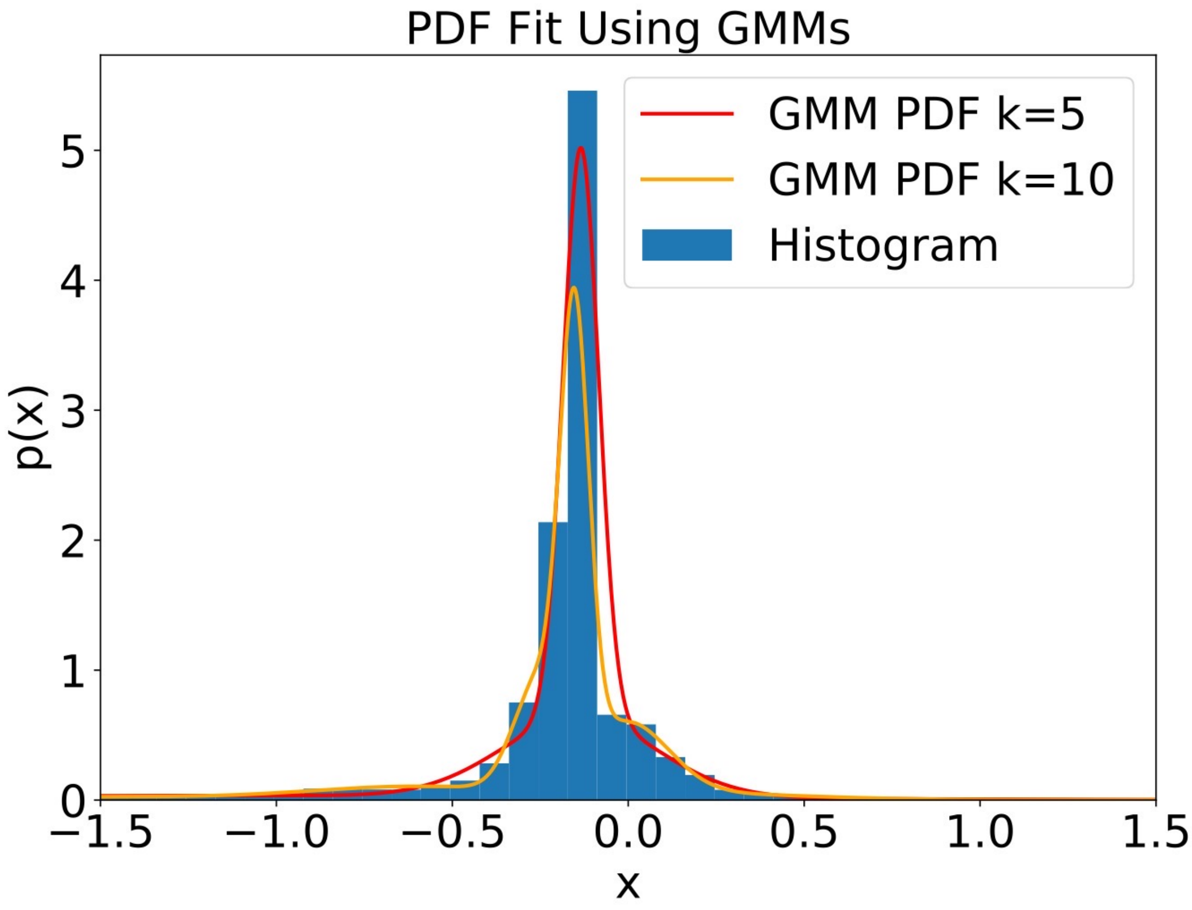
Unstable Gradients



Stabilized Gradients

- We compute PDF with Gaussian Mixture Models



Ours

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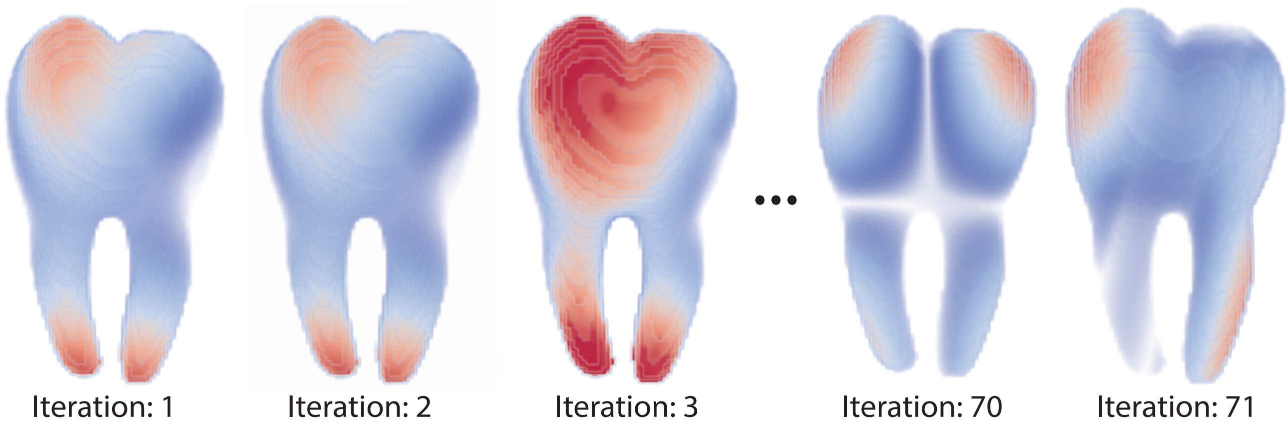
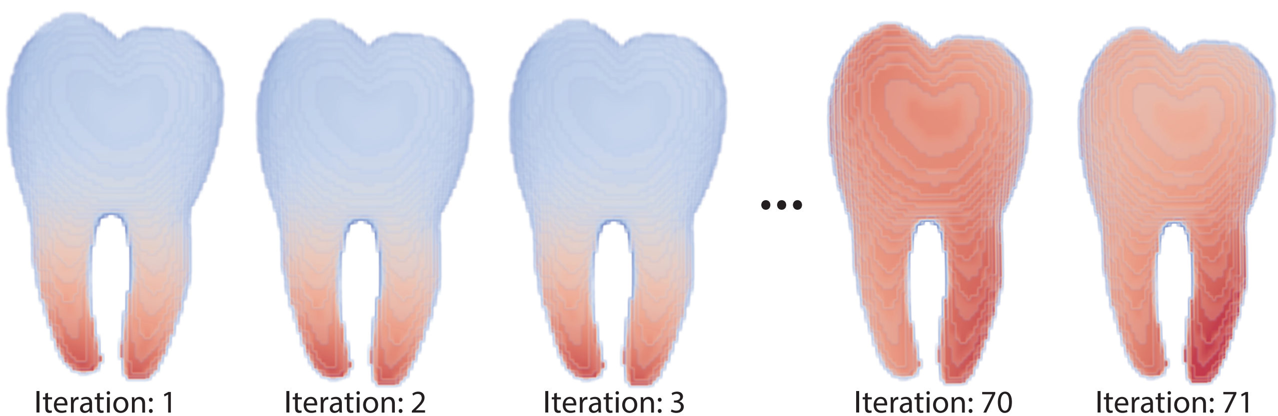
Stabilized Gradients

Optimized results



[Langlois et al. 2016]



Ours

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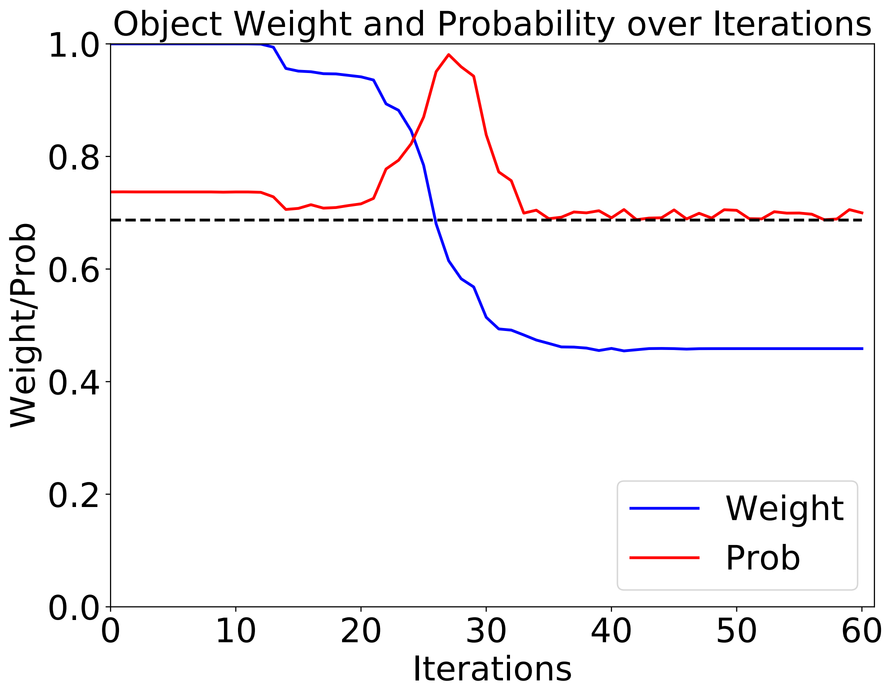
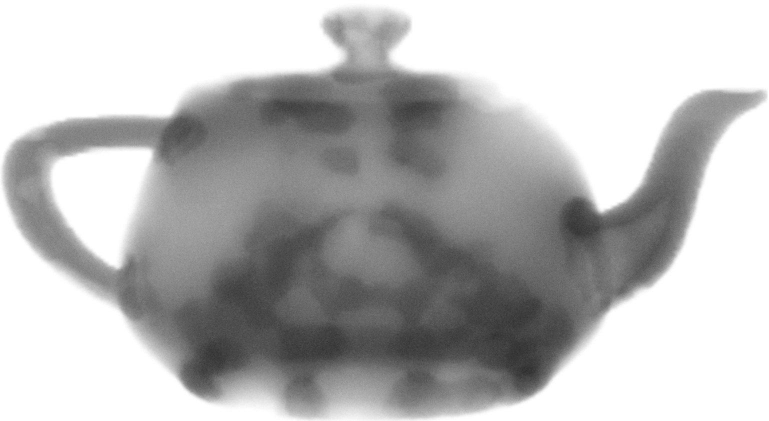
Local Minima

The objective function: $\min f(\boldsymbol{\omega}) = \sum_{e=1}^{m} \boldsymbol{\omega}_e$

The constraint function: $g(\boldsymbol{\omega}) = \Theta - P(s < 1) < 0$

$g(\boldsymbol{\omega})$ is extremely non-linear → Local minima



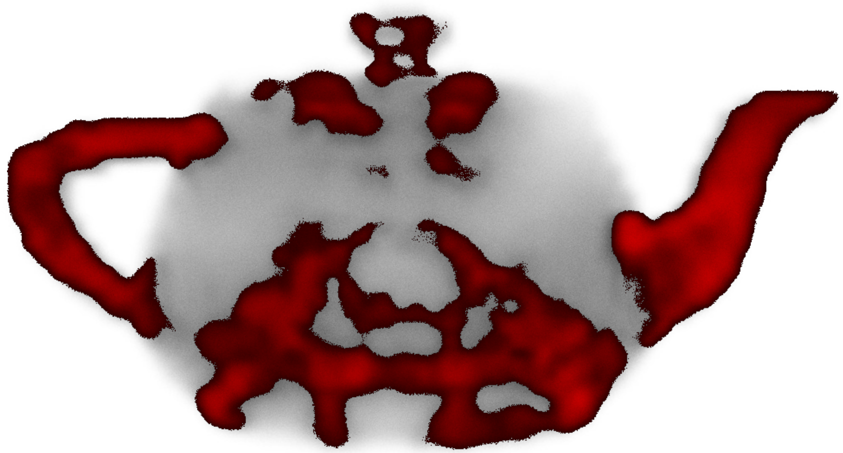


Iteration: 59

Reinforcement Structures

Intermediate results

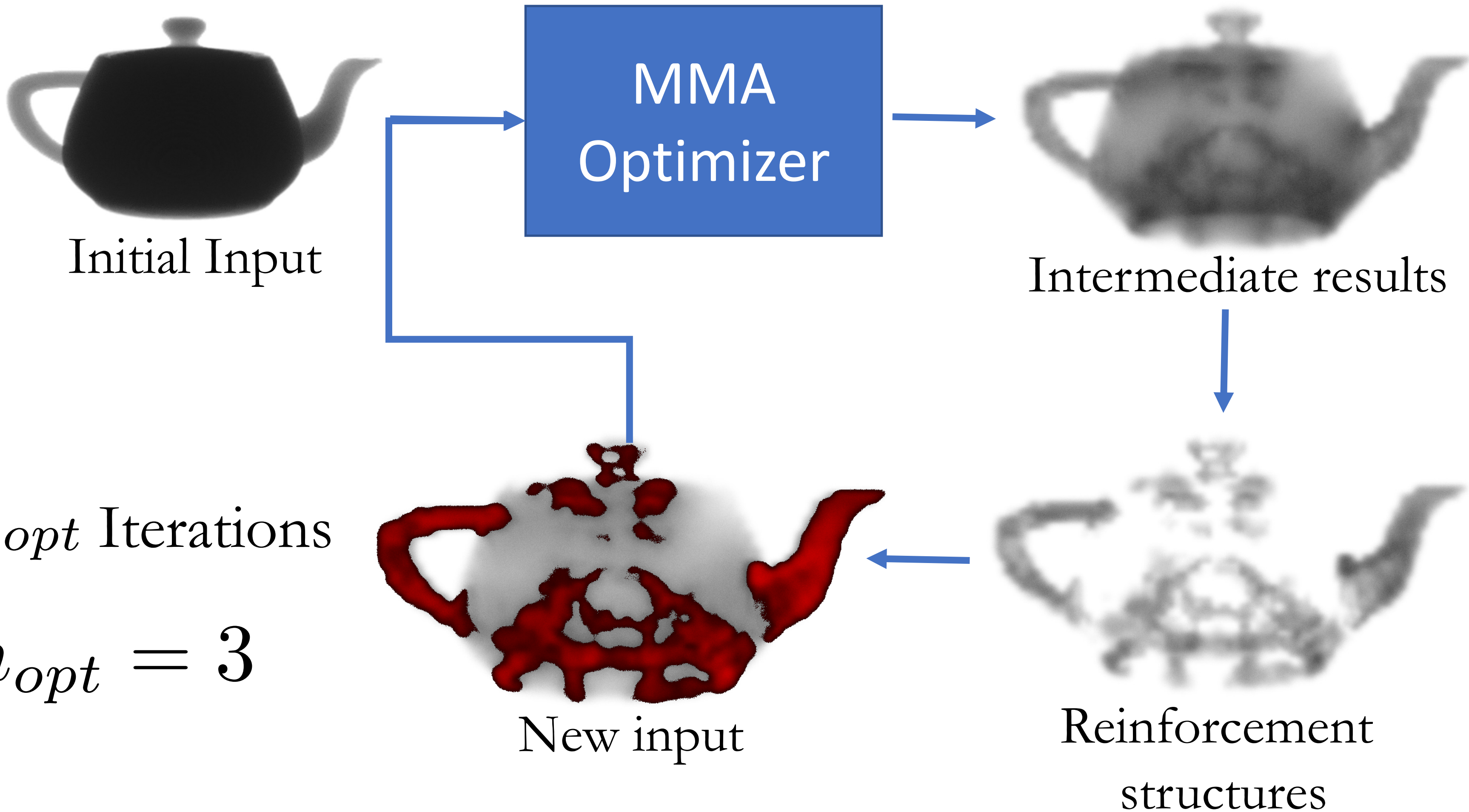
Reinforcement Structures



→ Constrained

Input for the the next optimization

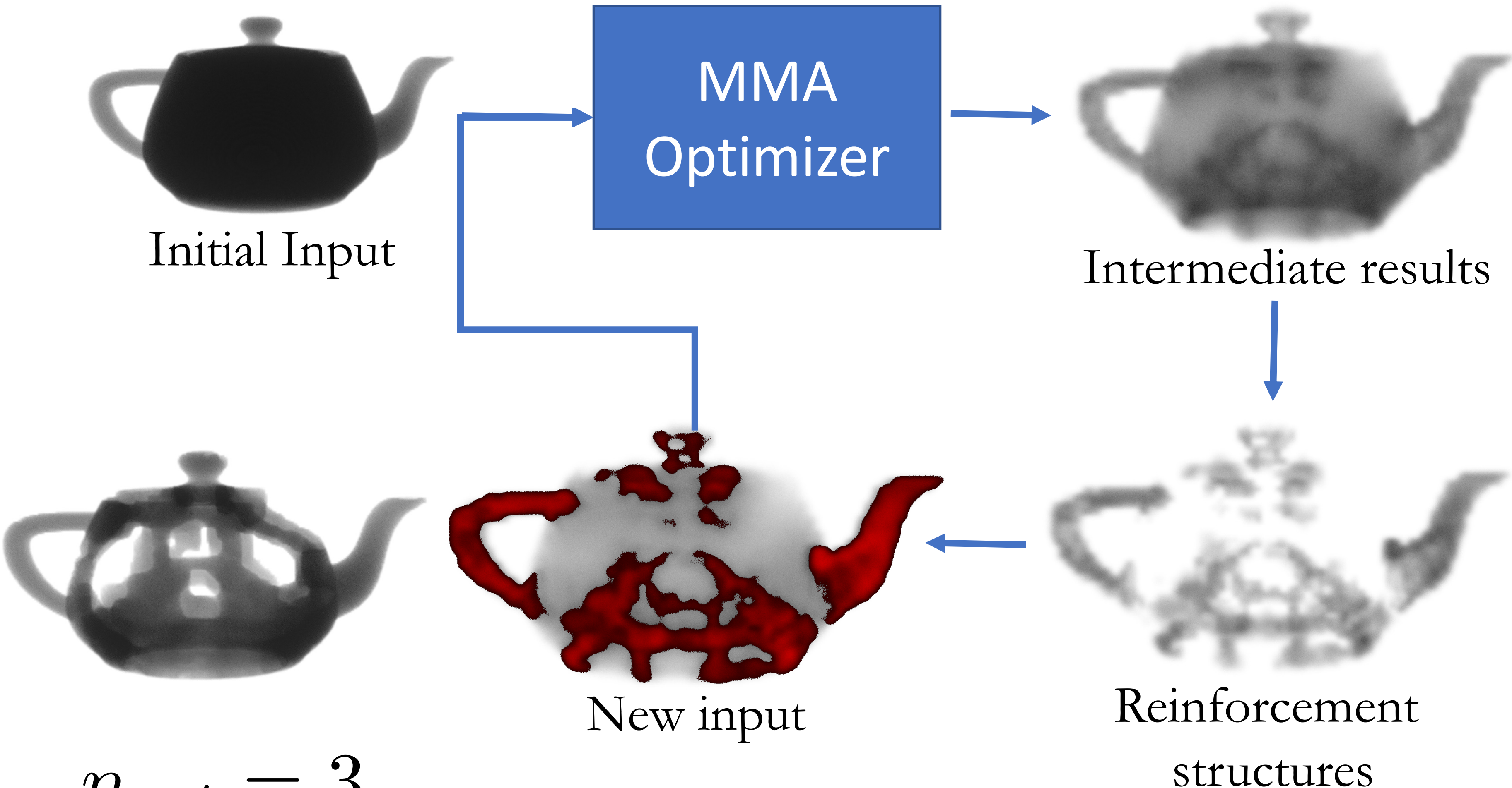
The Restart Strategy



n_{opt} Iterations

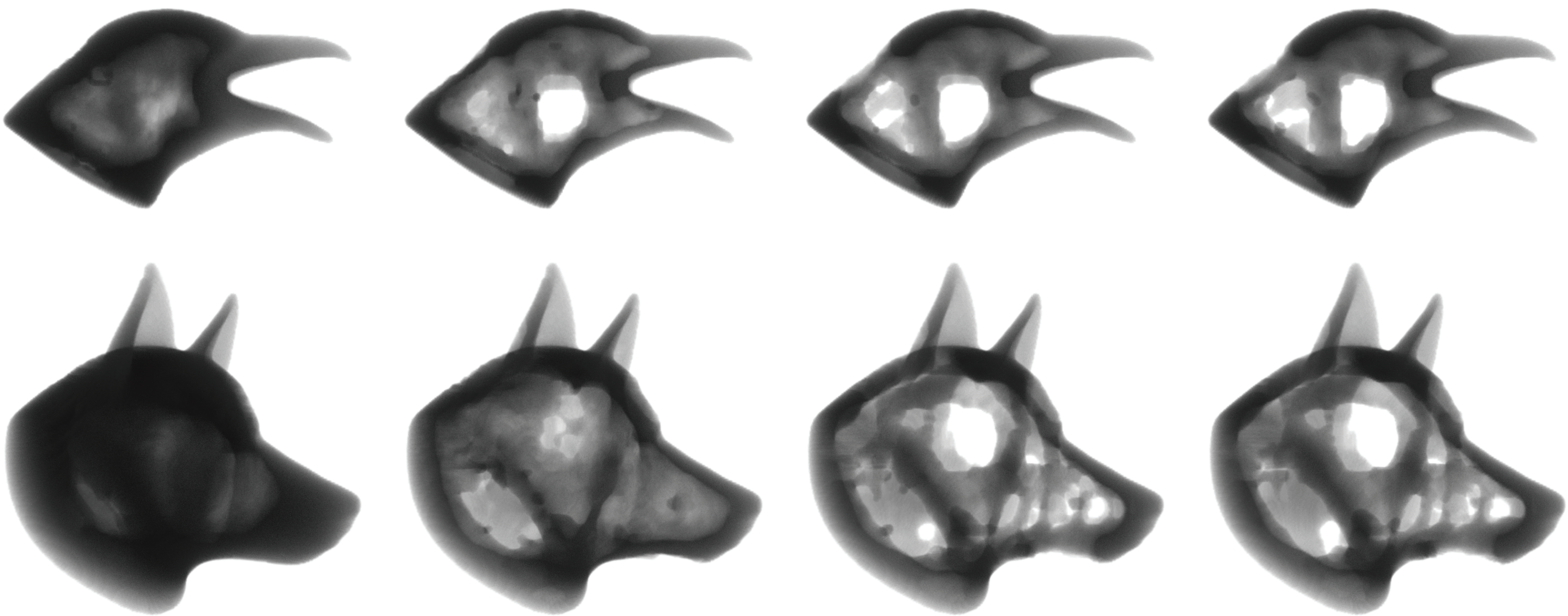
$$n_{opt} = 3$$

The Restart Strategy



$$n_{opt} = 3$$

Parameter Choice



$n_{opt}=1$ $n_{opt}=2$ $n_{opt}=3$ $n_{opt}=4$

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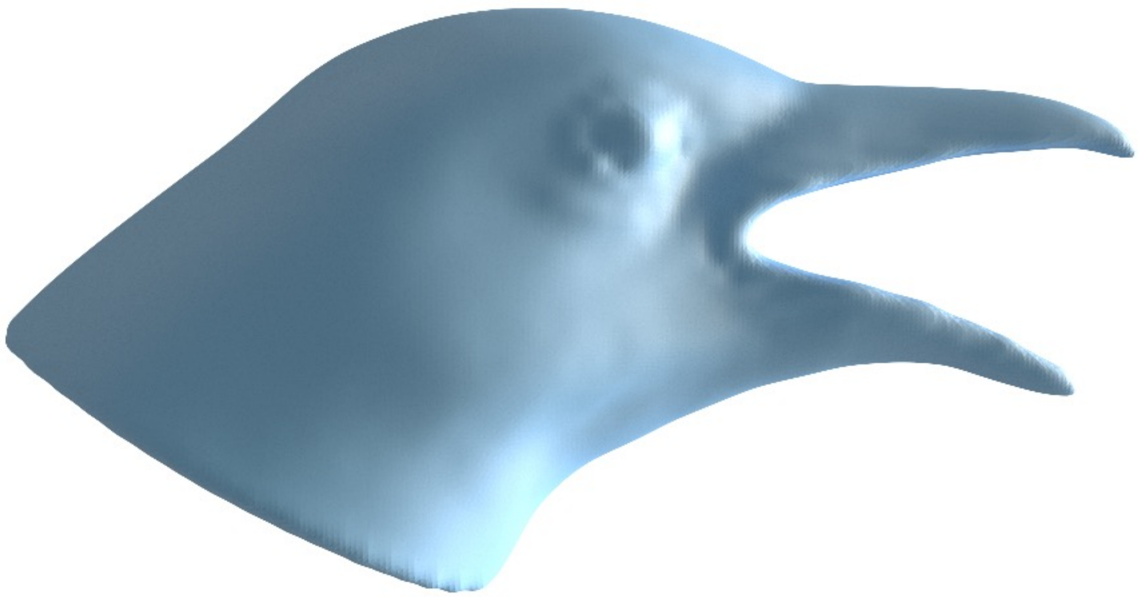
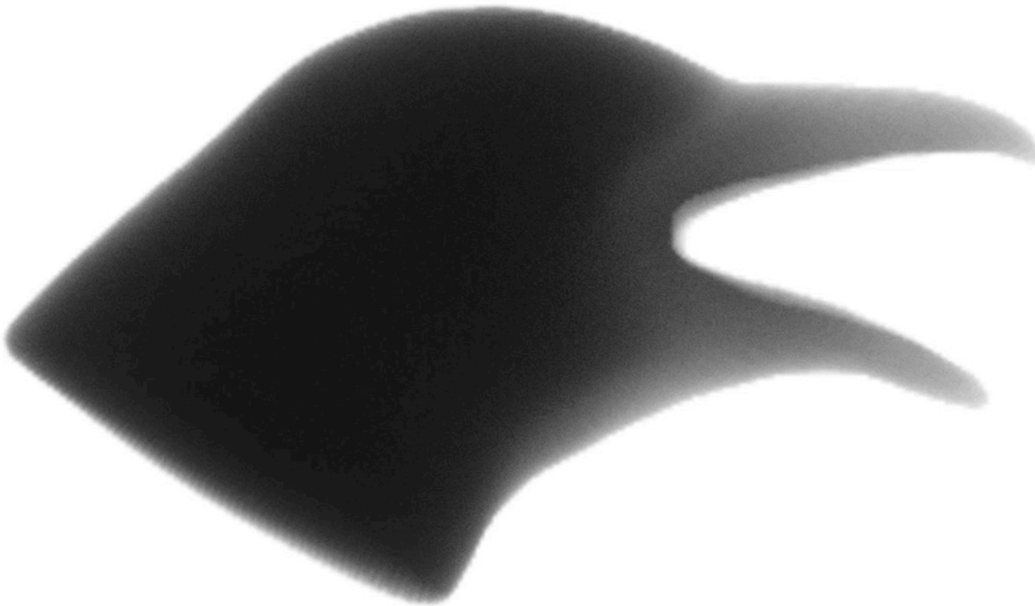
Raven



Resolution: $32 \times 64 \times 40$



Optimization iteration: 0

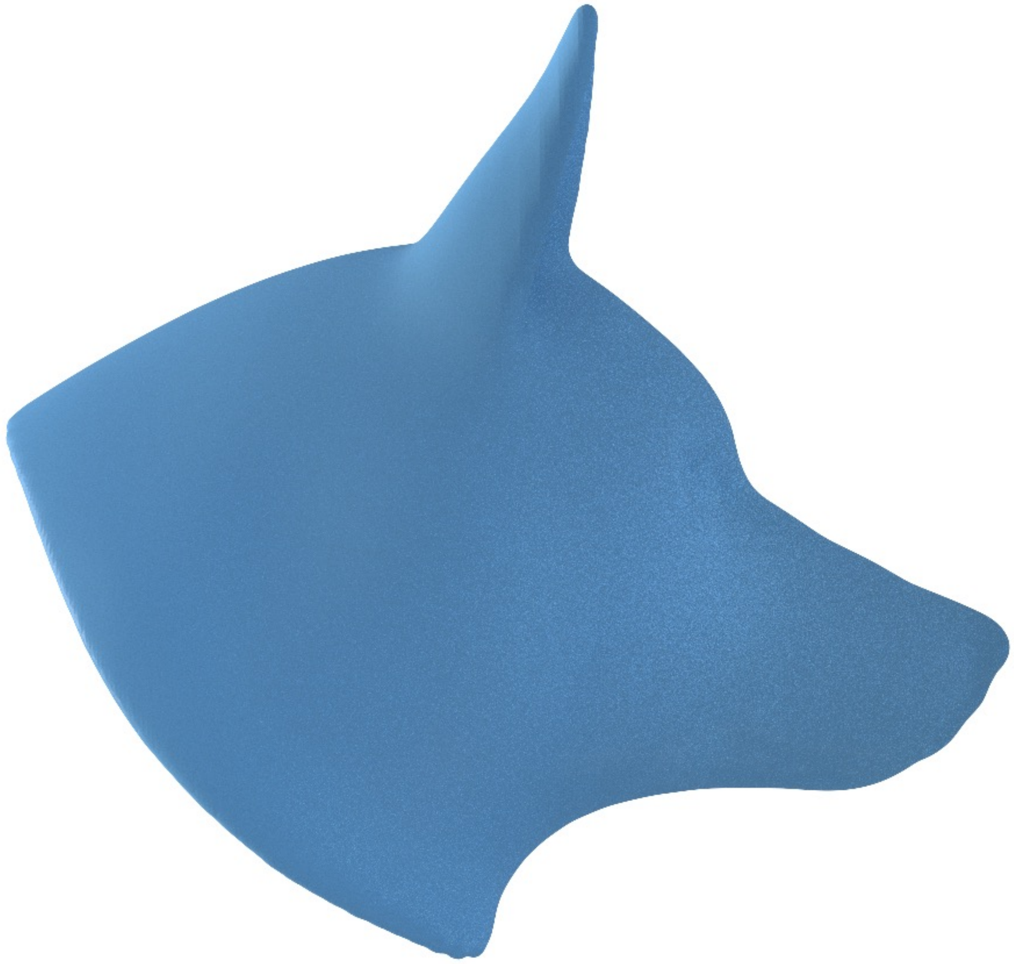
Time per iteration

Previous: 11.87 hrs

Ours: 5.46 minutes

130×

Dog



Resolution: $40 \times 64 \times 60$

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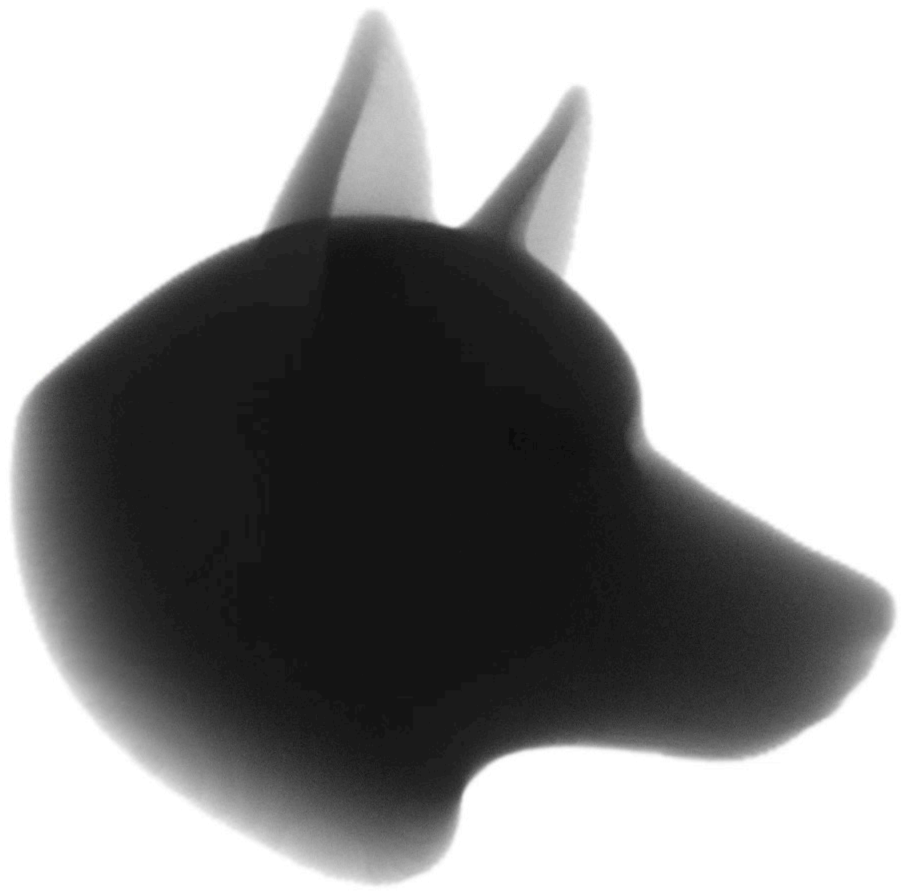

Time per iteration

Previous: 41.72 hrs

Ours: 7.68 minutes

326×

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Armadillo

Resolution: $56 \times 64 \times 52$

Optimization iteration: 0

Time per iteration

Previous: 61.42 hrs

Ours: 6.6 minutes

558×

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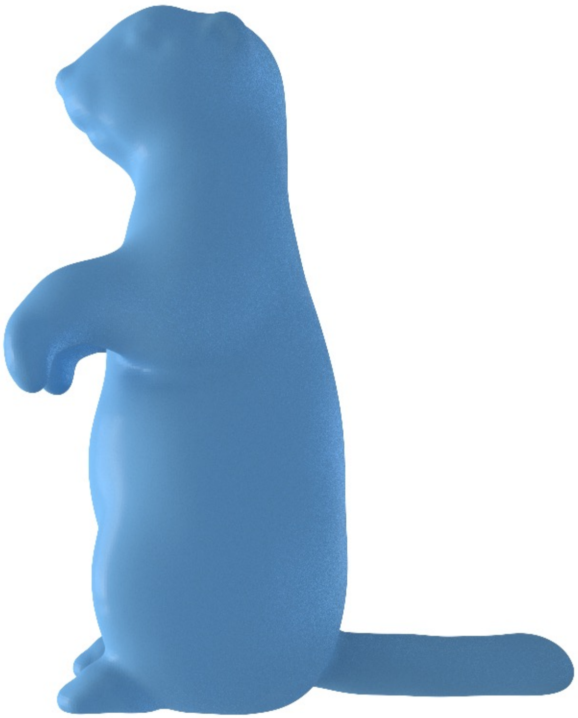
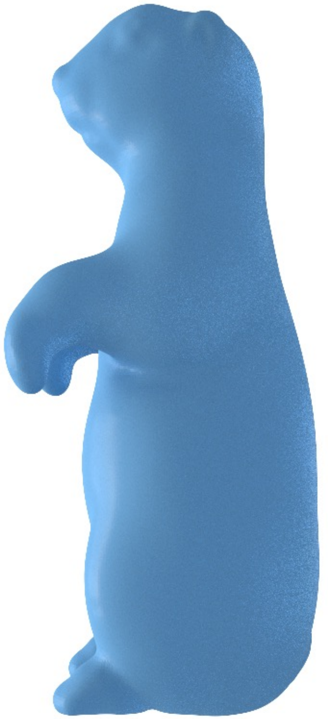
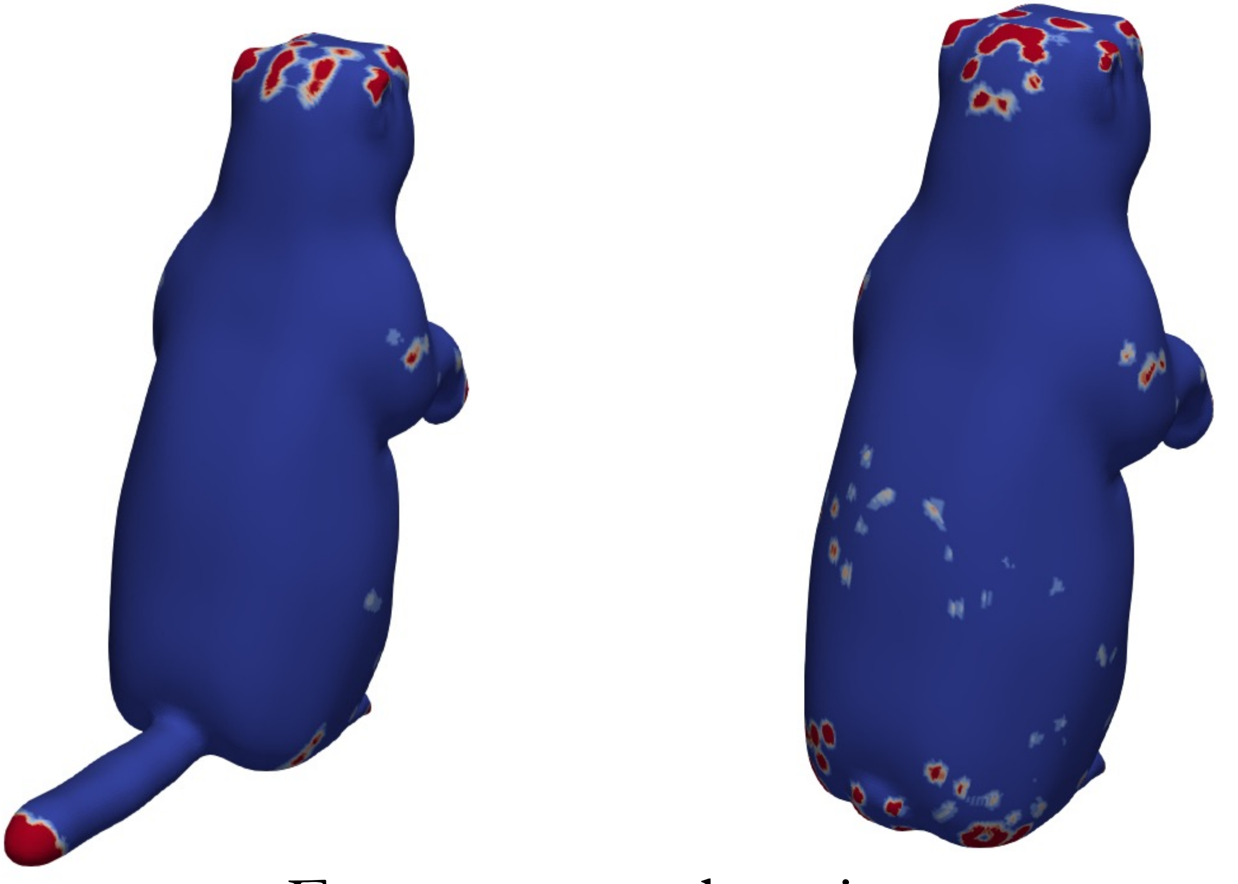
Adaptative to Real World Loading





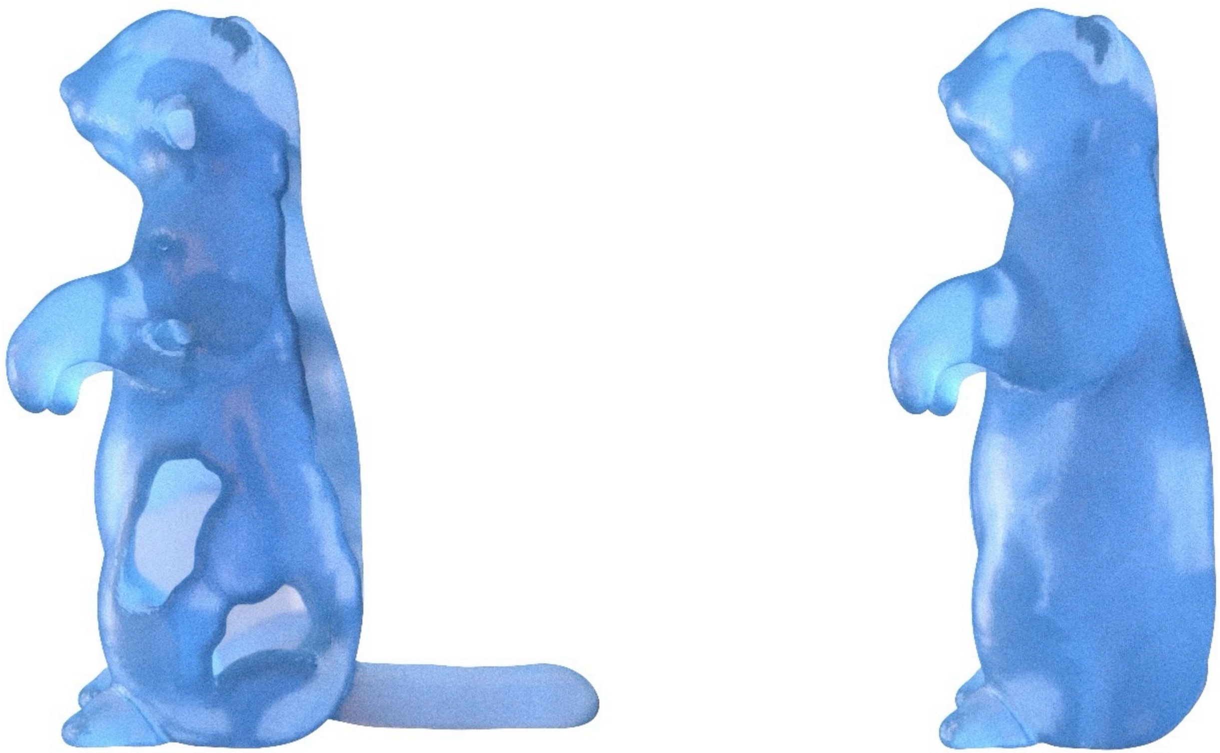
Initial Objects

Adaptative to Real World Loading



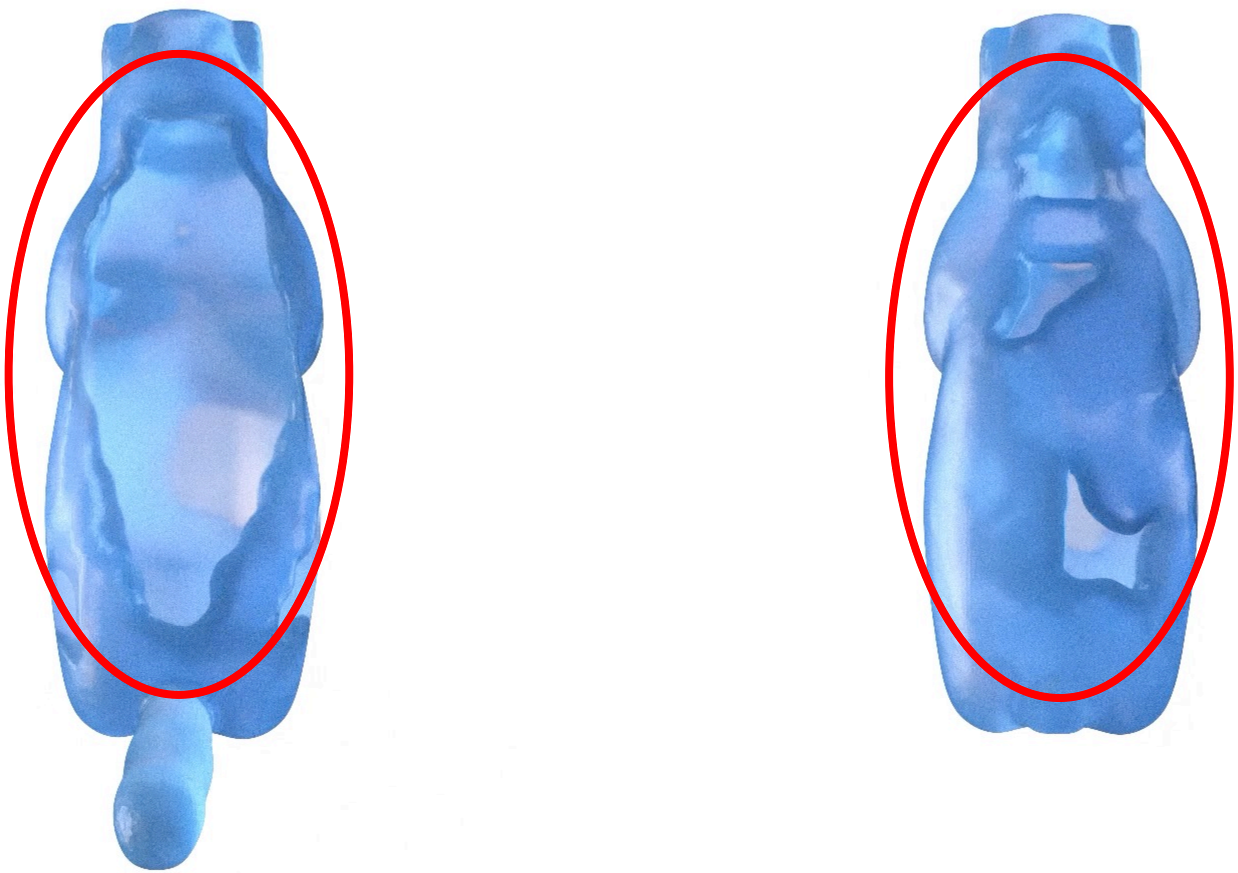
Force contact locations

Adaptative to Real World Loading



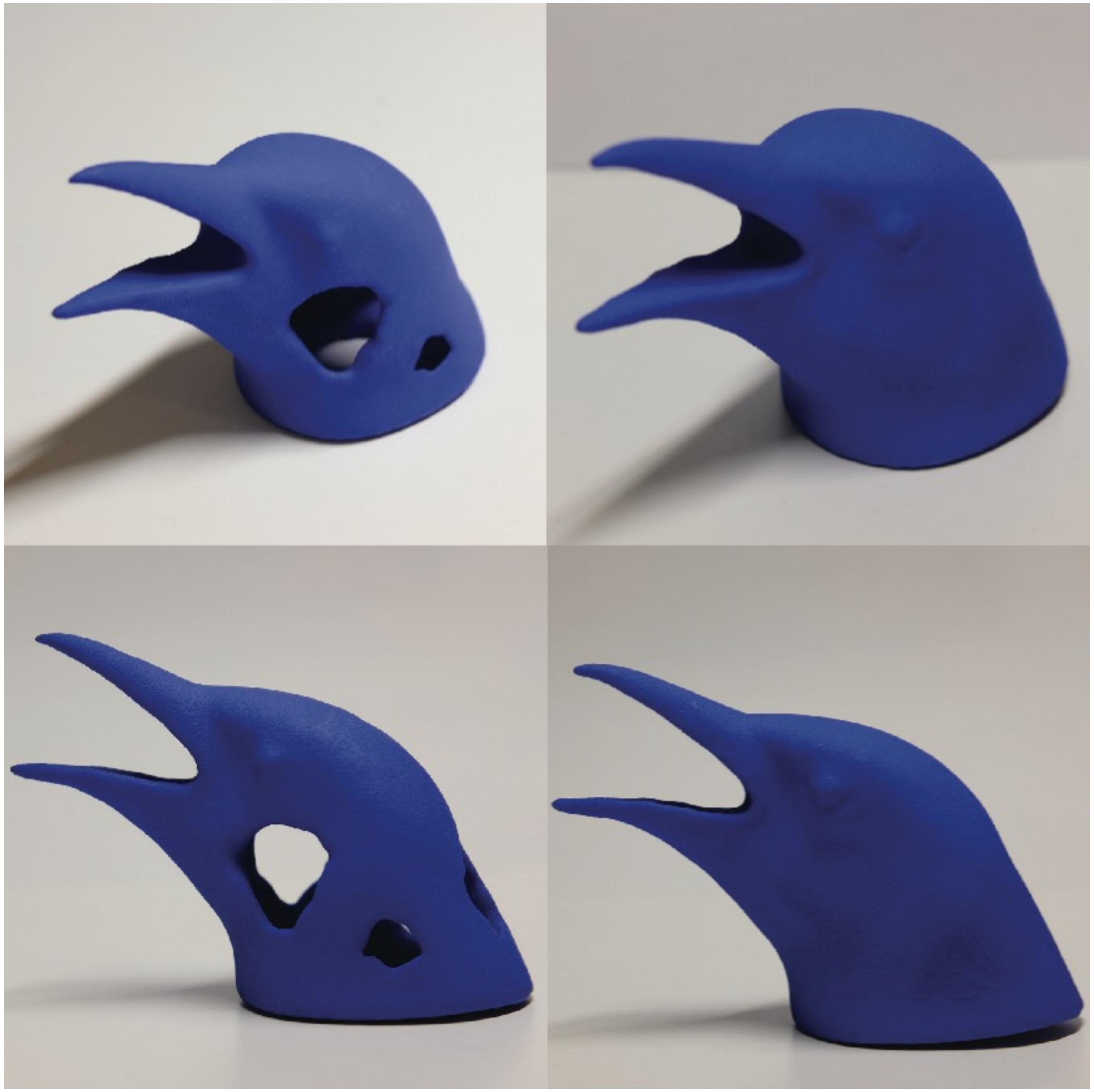
Optimized results

Adaptative to Real World Loadings



Optimized results

Physical Validation



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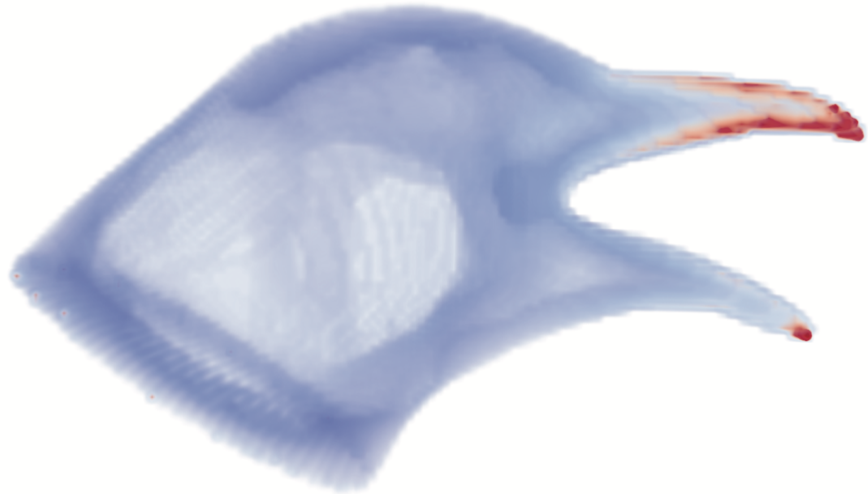
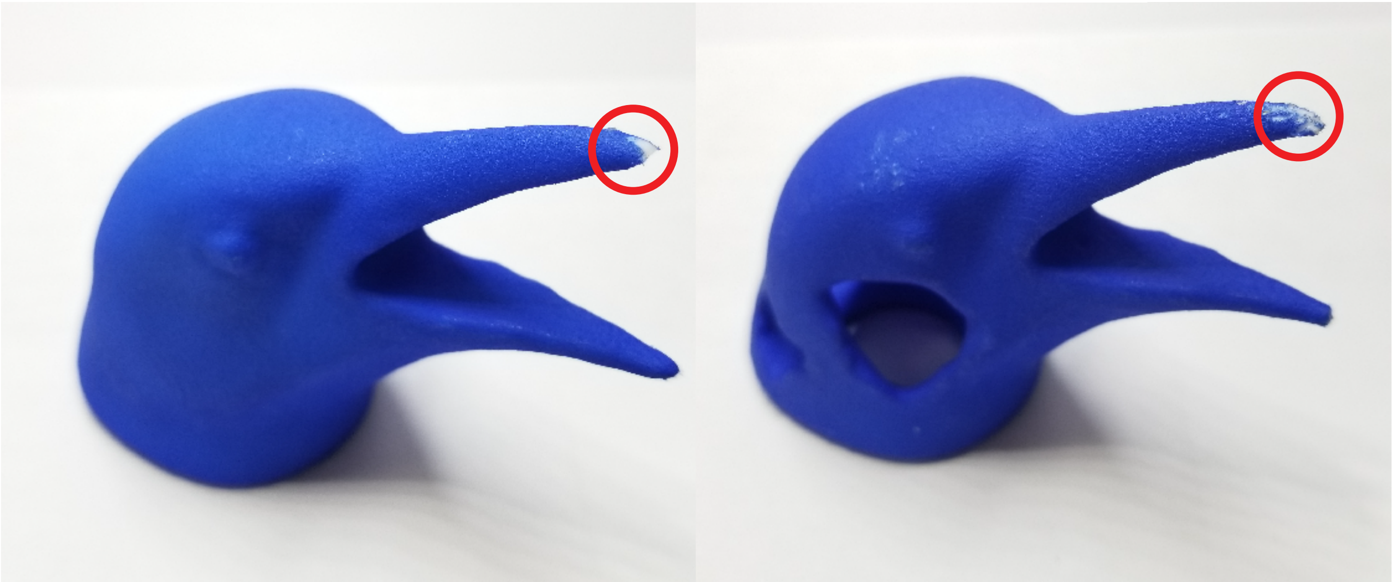
Physical Validation

Physical Validation





Outline

- Previous work
- Stochastic Structural Optimization
- Our methods
  - Faster gradients computations
  - Robust gradients
  - A constrained restart strategy
- Results
- Contributions and future work

Contributions

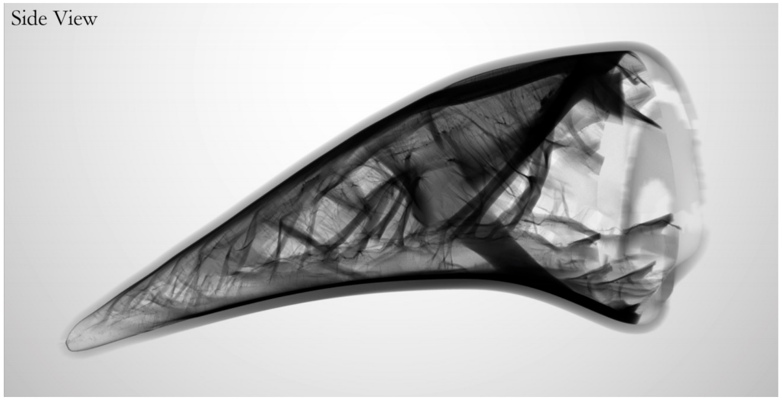
- Fast and Robust Stochastic Structural Optimization
  - Asymptotically faster $O(n^2) \rightarrow O(n)$
  - Robust, stable probability gradients
  - A constrained restart strategy

Limitations

- Force contact locations are fixed
- Expensive gradient computation
- Requires multiple optimization passes

Future Work

- Sparse optimization
- Identify the reinforcement structures on the fly
- Incorporate shape change at contact locations



[Liu et al. 2018]

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Thank You